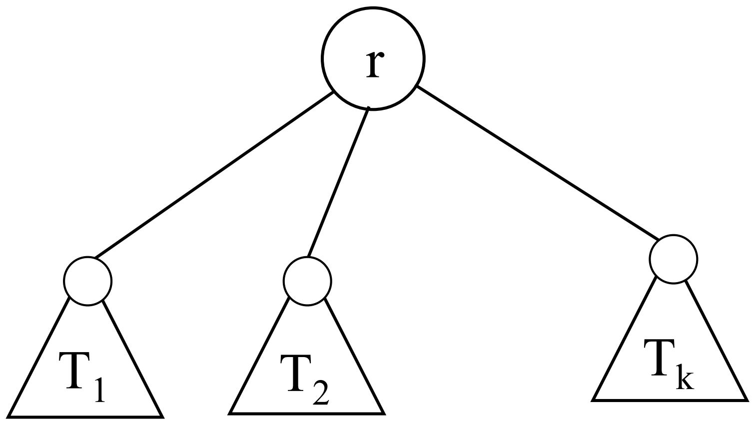
Data Structure:
Tree

tree

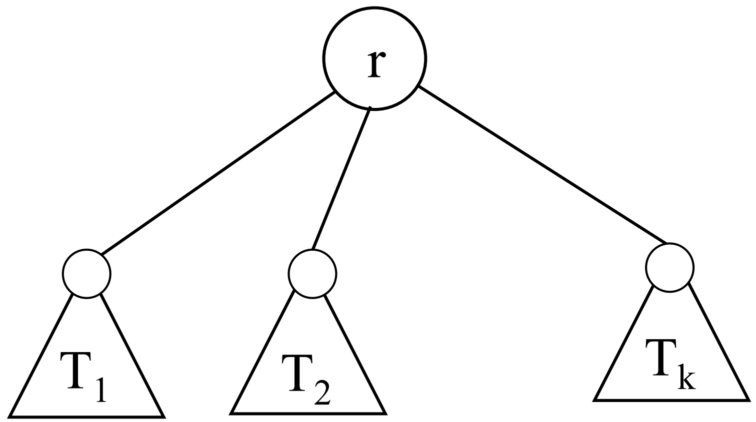
- a collection of nodes connected by edges without a cycle
- by recursive definition:
  - an empty tree or
  - a root r and subtrees $T_1, T_2, \ldots, T_k$ (disjoint sets) each of whose roots are connected to r by an edge

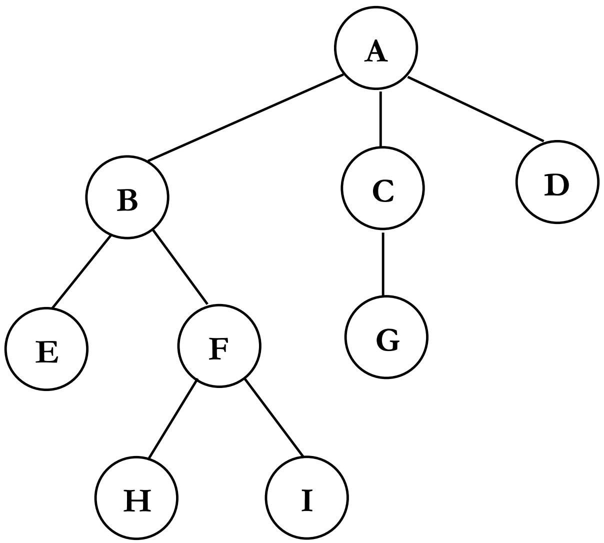


recursive definition of tree

tree

- Each root of $T_1, T_2, \ldots, T_k$ is a *child* of *r*, and *r* is the *parent* of each root.
- The roots of the subtrees are *siblings* of one another
- If there is an order among the T_i's, the tree is an *ordered tree*.
- The *degree* of a node is the number of children it has.
- The *degree* of a tree is the maximum degree of the nodes.
- A *leaf* is a node of degree 0.

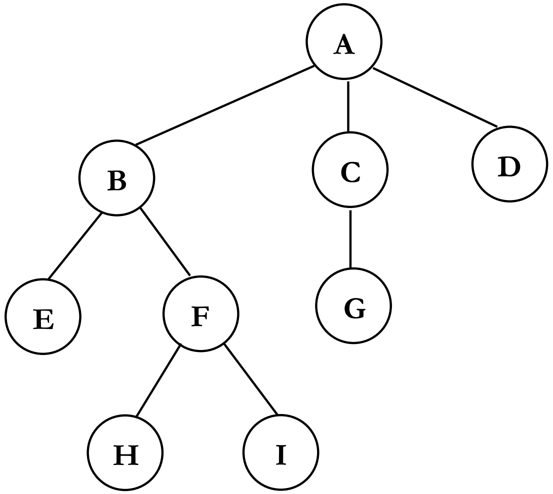




an example of tree

tree

- *path* between two nodes is a sequence of nodes $n_1, n_2, \ldots n_k$, such that n_i is a parent of n_{i+1}
- *length* of a path is the number of edges on the path (the path $n_1, n_2, \ldots n_k$: length k-1)
- *depth (level)* of a node is the length of the (unique) path from the root to that node (root: level 0)
- *height* of a node is the length of the longest path from that node to a leaf (leaf: height 0)
- the height of a tree is the height of the root

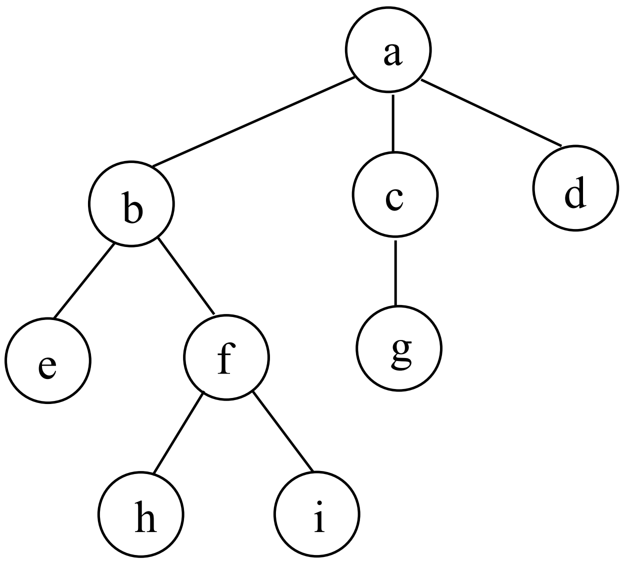


representation of tree

- for any node x, there exists exactly one path from the root to x?
- tree can be empty with no node?
- how many edges are in a tree with n nodes?

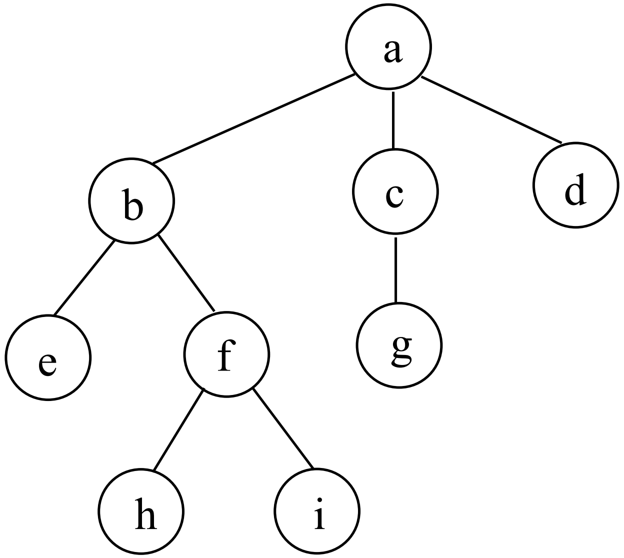
representation of tree

- how can we implement a tree?
  - linked list?
  - can we have pointers for the children nodes?
  - can we have fixed number of pointers to represent a tree?
    - for a tree of fixed number of degree?
    - else?



left child-right sibling representation

- every node has at most one leftmost child and at most one closet right sibling



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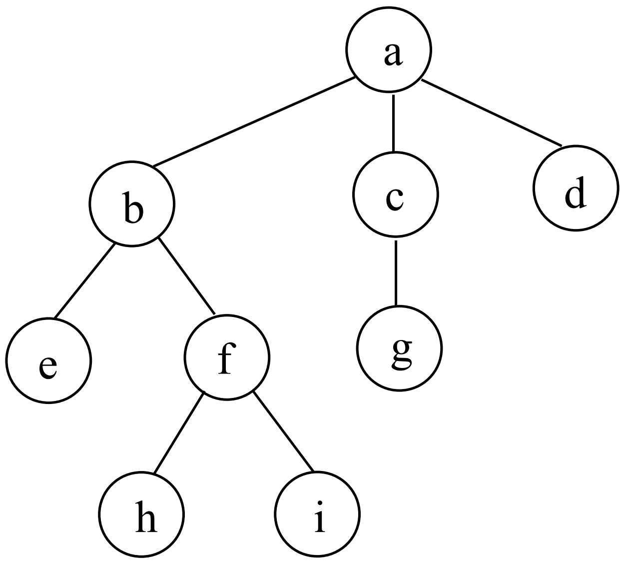
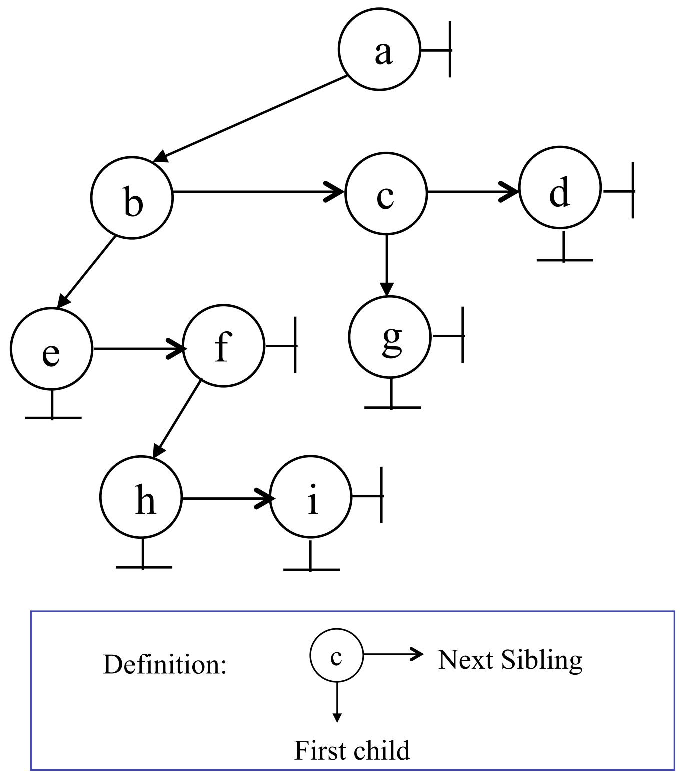
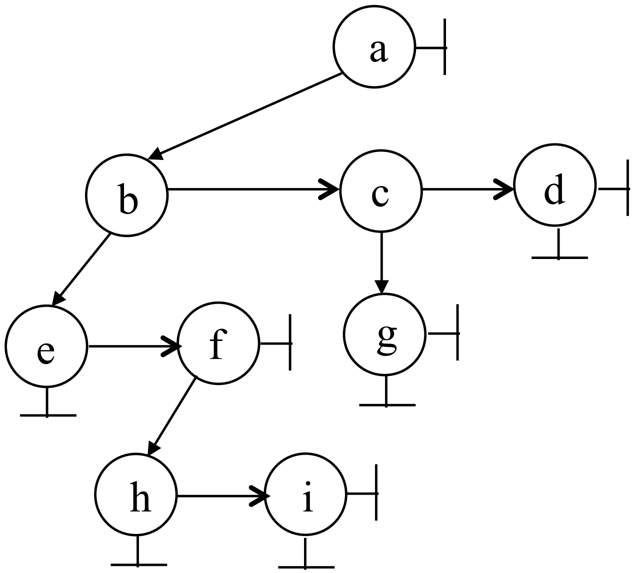
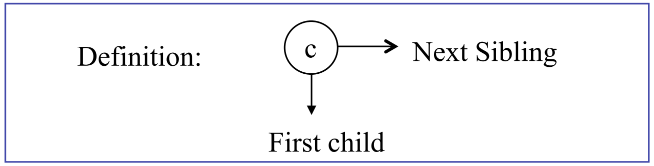




left child- right sibling representation






```
struct TreeNode{
    ElementType Element;
    PtrToNode FirstChild;
    PtrToNode NextSibling;
    };
typedef struct TreeNode *PtrToNode;
```

binary tree

- a finite set of nodes that is either
  - i) empty or
  - ii) a root node and two disjoint binary trees
- the tree on the left and the tree on the right are different

binary tree

- the maximum number of nodes on level i of a binary tree is 2^i, $i>=0$

  the proof by induction
  - base: for the root at level $i=0$, $2^0 = 1$
  - induction hypothesis: assume that the maximum number of nodes on level $i-1 > 0$, 2^{i-1}
  - induction step: on level i,

    $2 * (\text{ the maximum number of nodes on level } i-1) = 2 * 2^{i-1} = 2^i$

- the maximum number of nodes in a binary tree of depth k is $2^{k+1}-1$, $k>=0$

binary tree

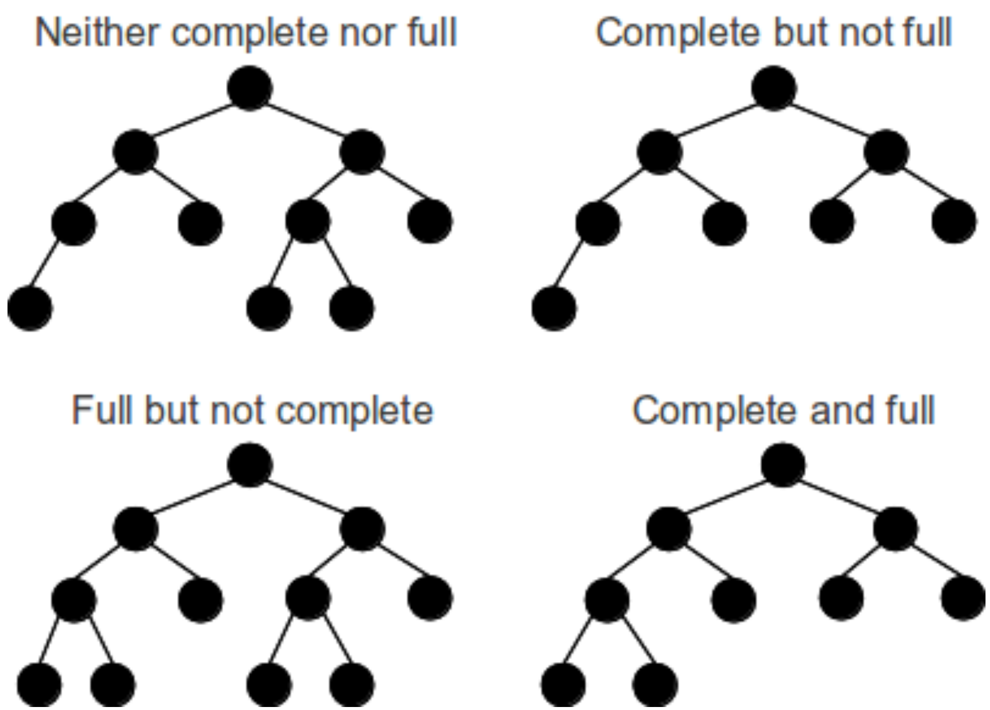
- For any nonempty binary tree T, if n_0 is the number of leaf nodes, and n_2 is the number of nodes of degree 2, then $n_0 = n_2 + 1$

$n = n_0 + n_1 + n_2$, n_i is the number of nodes with i degree
n is the number of nodes in the tree

$n = B + 1 = n_1 + 2n_2 + 1$, B is the number of branches (edge)

binary tree

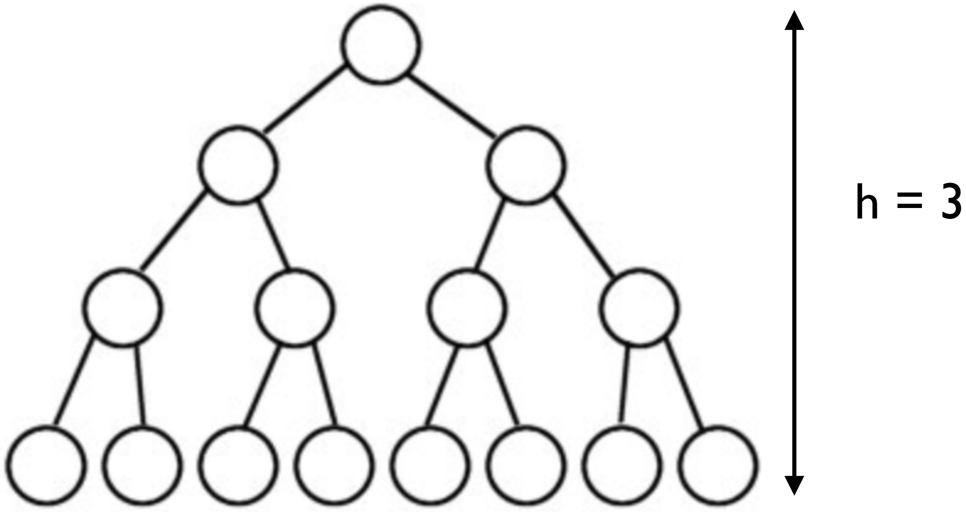
- full binary tree is a binary tree in which every node has 0 or 2 children
- complete binary tree is a binary tree in which every level, except the last, is completely filled and the last level has all its nodes to the left side



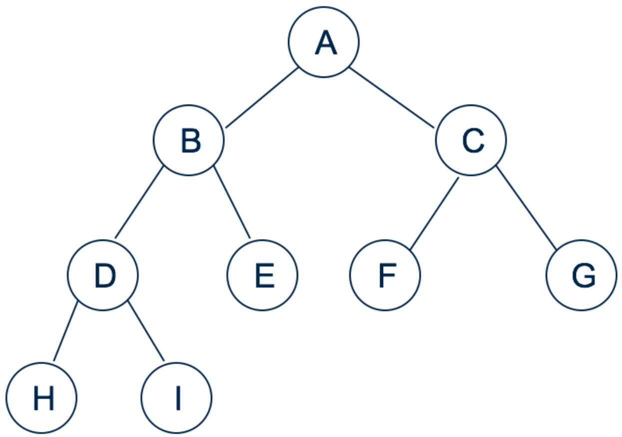
binary tree

- perfect binary tree of height h is a binary tree of height h having $2^{h+1} - 1$ nodes, (h >=0)
- the max number of nodes in the complete binary tree (height h) is $2^{h+1} - 1$

$$2^0 + 2^1 + \dots + 2^h = (2^{h+1} - 1)/(2 - 1) = 2^{h+1} - 1$$



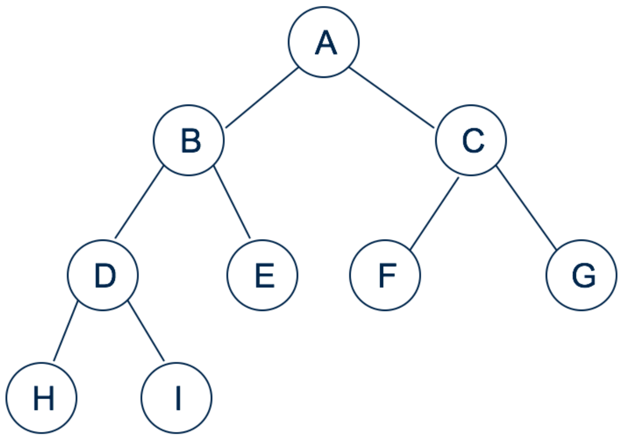
binary tree: array representation



[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

binary tree: array representation

- if a complete binary tree with n nodes (i is the index) is represented sequentially,
  - leftChild(i) is at 2i for 2i <=n
  - rightChild(i) is at 2i + 1 for 2i + 1 <= n
  - parent(i) is at $\lfloor i/2 \rfloor$ for i >1



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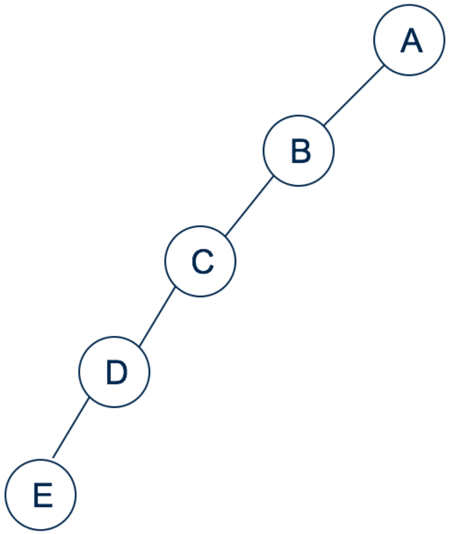


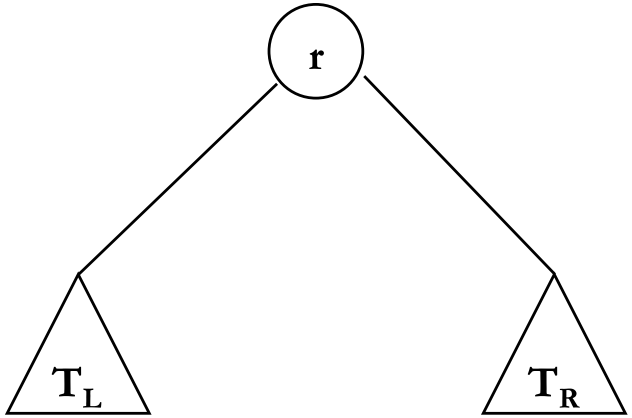
[1]	A
[2]	B
[3]	-
[4]	C
[5]	-
[6]	-
[7]	-
[8]	D
[9]	-
.	.
.	.
.	.
[16]	E

binary tree: linked list representation

- a tree in which each node has no more than 2 children (left subtree and right subtree)

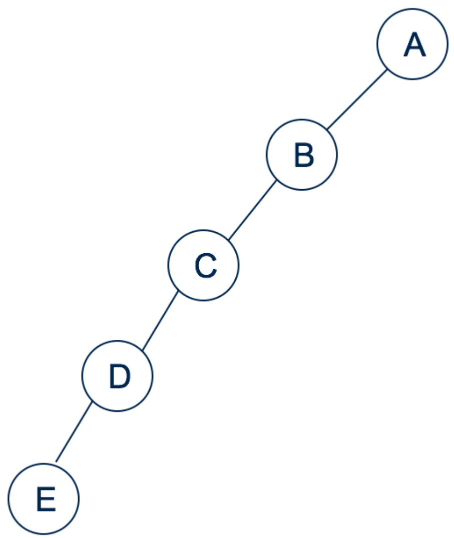
```
struct TreeNode
{
      ElementType   Element;
      Tree          Left;
      Tree          Right;
};

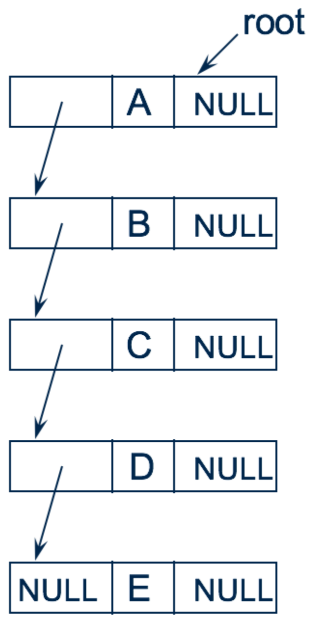
typedef struct TreeNode* PtrToNode;
typedef struct PtrToNode Tree;
```

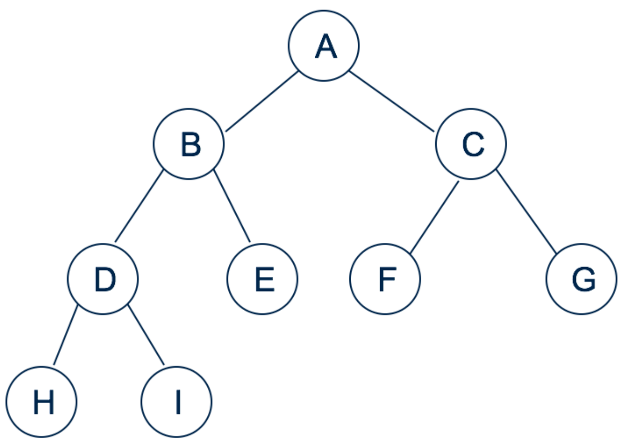


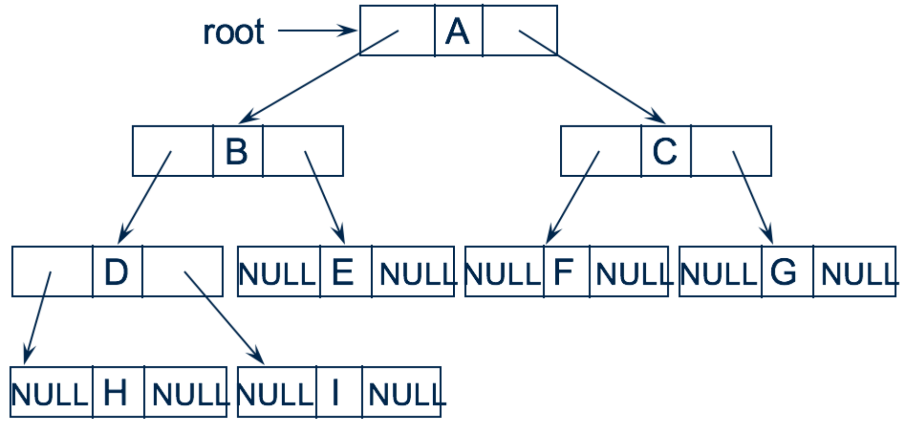
Left	Element	Right

binary tree: linked list representation



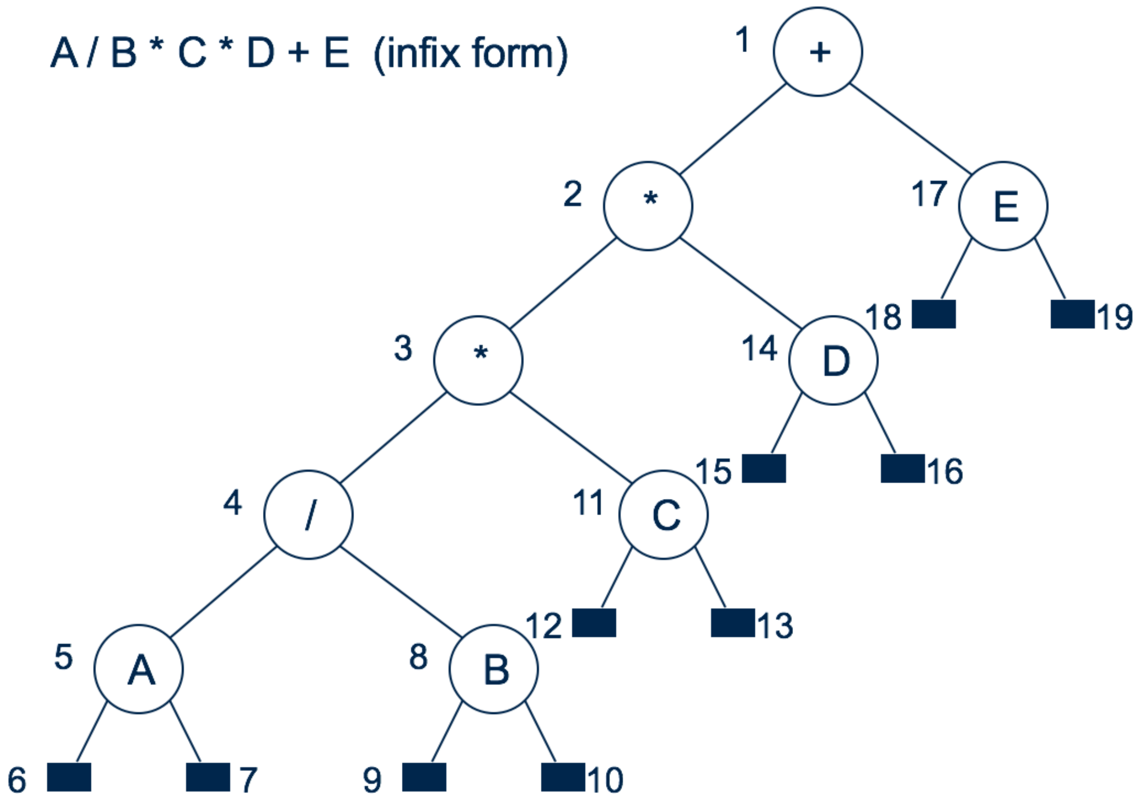






application of binary tree

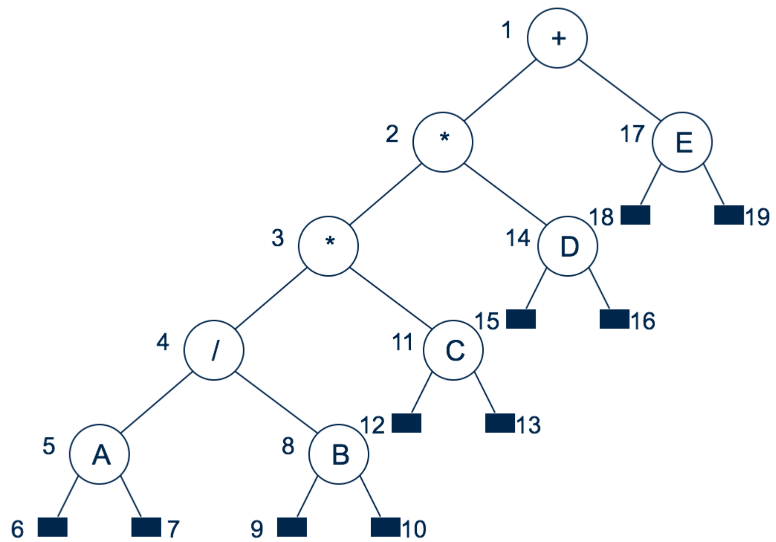
- *Expression Tree*: intermediate representation for expressions used by the compiler



tree traversal

- inorder traversal

```
void inorder(Tree ptr) {
    if(ptr) {
        inorder(ptr->left_child);
        printf("%d",   ptr->data);
        inorder(ptr->right_child);
    }
}
```



call of inorder	value in root	action	call of inorder	value in root	action
1	+		11	C	
2	*		12	NULL	
3	*		11	C	printf
4	/		13	NULL	
5	A		2	*	printf
6	NULL		14	D	
5	A	printf	15	NULL	
7	NULL		14	D	printf
4	/	printf	16	NULL	
8	B		1	+	printf
9	NULL		17	E	
8	B	printf	18	NULL	
10	NULL		17	E	printf
3	*	printf	19	NULL	

tree traversal

```
void preorder(Tree ptr) {
    if(ptr) {
        printf("%d",   ptr->data);
        preorder(ptr->left_child);
        preorder(ptr->right_child);
    }
}
```

```
void postorder(Tree ptr) {
    if(ptr) {
        postorder(ptr->left_child);
        postorder(ptr->right_child);
        printf("%d",   ptr->data);
    }
}
```

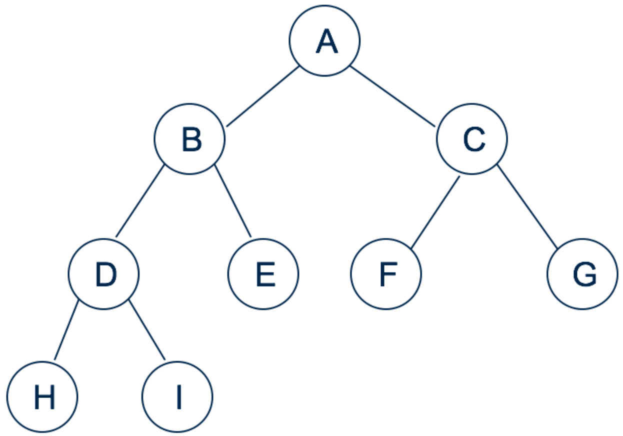
tree traversal

- iterative in-order traversal using stack

```
void iterInorder (Tree node)   {

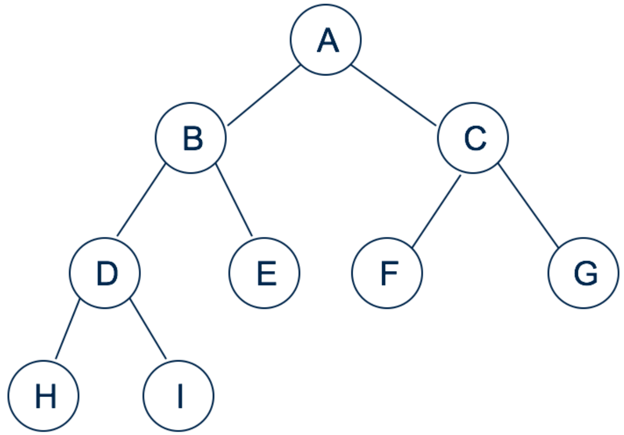
        int top = -1
        Tree stack[MAX_SIZE];
        for (; ;)   {
                for (; node; node = node -> leftChild)
                        push(node);

                node = pop();               // pop parent
                if (!node)    break;
                printf("%d", node -> data);
                node = node -> rightChild;
        }
}
```



tree traversal

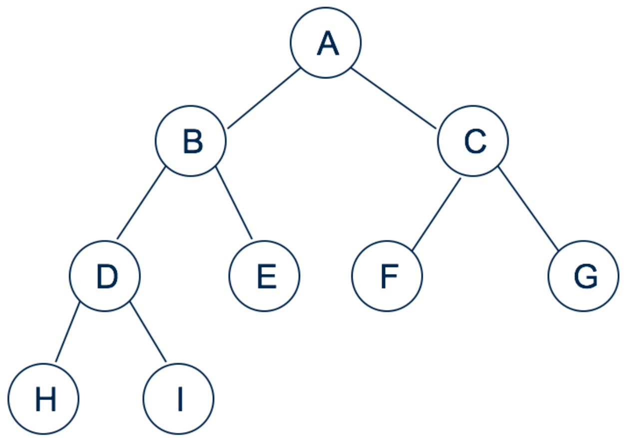
- level-order traversal



tree traversal

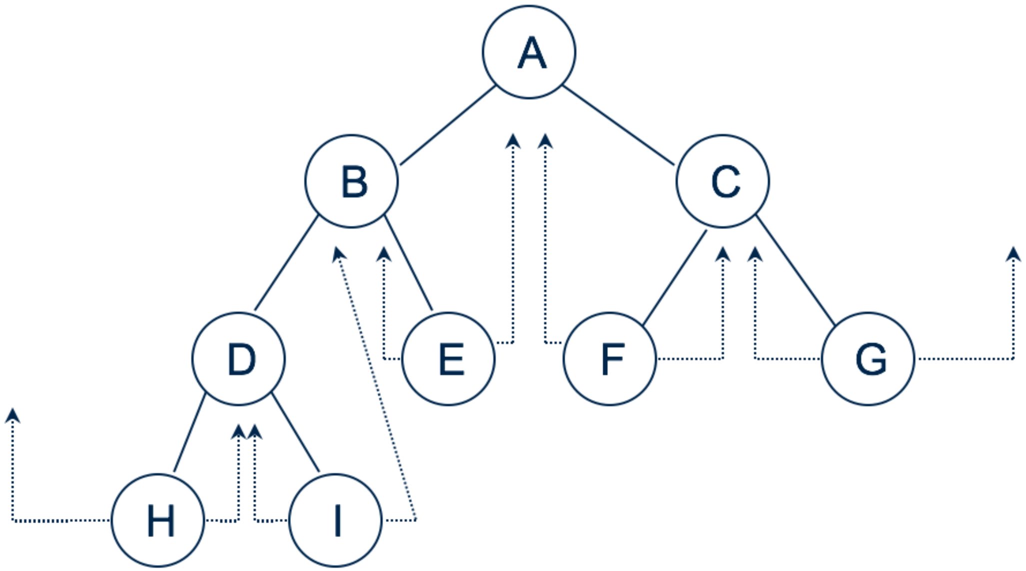
- level-order traversal

```
void levelOrder (Tree ptr)   {
        int front = 0;
        int rear = -1;
        Tree queue[MAX];
        if (!ptr)    return;
        addq(ptr);
        for (; ; )  {
                ptr = deleteq();
                if (ptr)  {
                        printf("%d", ptr->data);
                        if (ptr -> leftChild)
                                addq(ptr -> leftChild);
                        if (ptr -> rightChild)
                                addq(ptr -> rightChild);
                }
                else break;
        }
}
```



threaded binary trees

- there are n+1 null links out of 2n total links
- replace the null links by pointers, called threads to other nodes in the tree
  - if ptr -> leftChild is null, replace the null with a pointer to the node that would be visited before ptr in an in-order traversal
  - if ptr -> rightChild is null, replace the null with a pointer to the node that would be visited after ptr in an in-order traversal



threaded binary trees

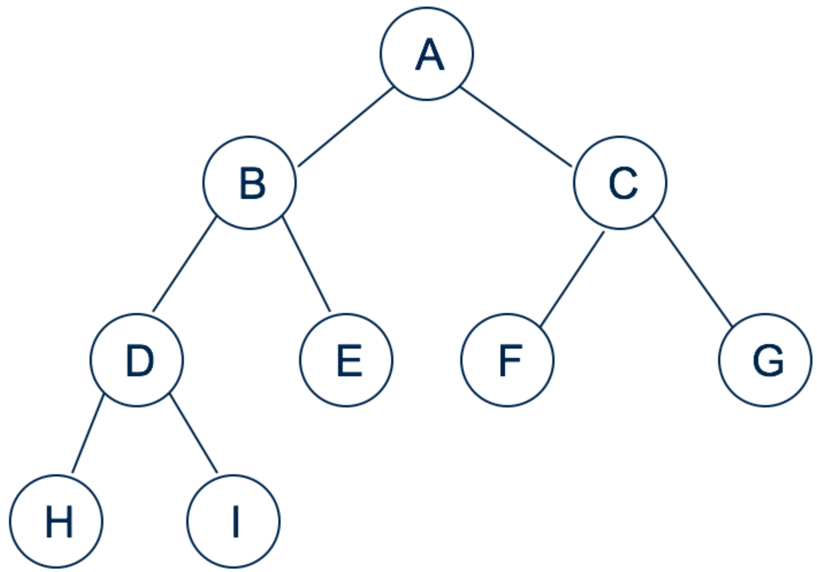
- How to distinguish actual pointers and threads?

  →add two additional fields to the node structure

  - if ptr->left_thread = true, ptr->left_child contains thread
  - if ptr->left_thread = false, ptr->left_child contains a pointer to the left child

```
typedef struct threaded_tree *threaded_ptr;
typedef struct threaded_tree {
        short int left_thread;
        threaded_ptr left_child;
        char data;
        threaded_ptr right_child;
        short int right_thread;
};
```

threaded binary trees



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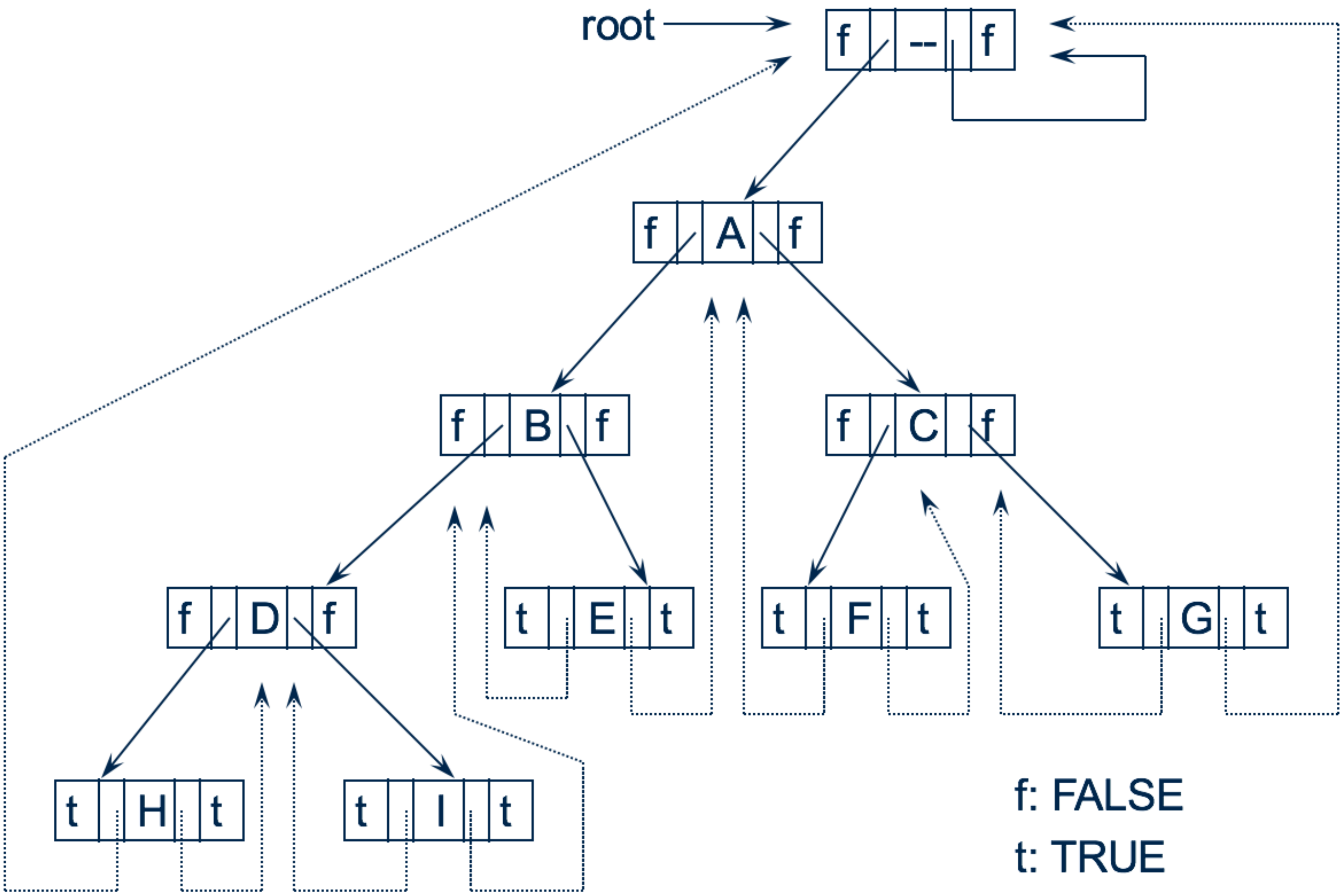
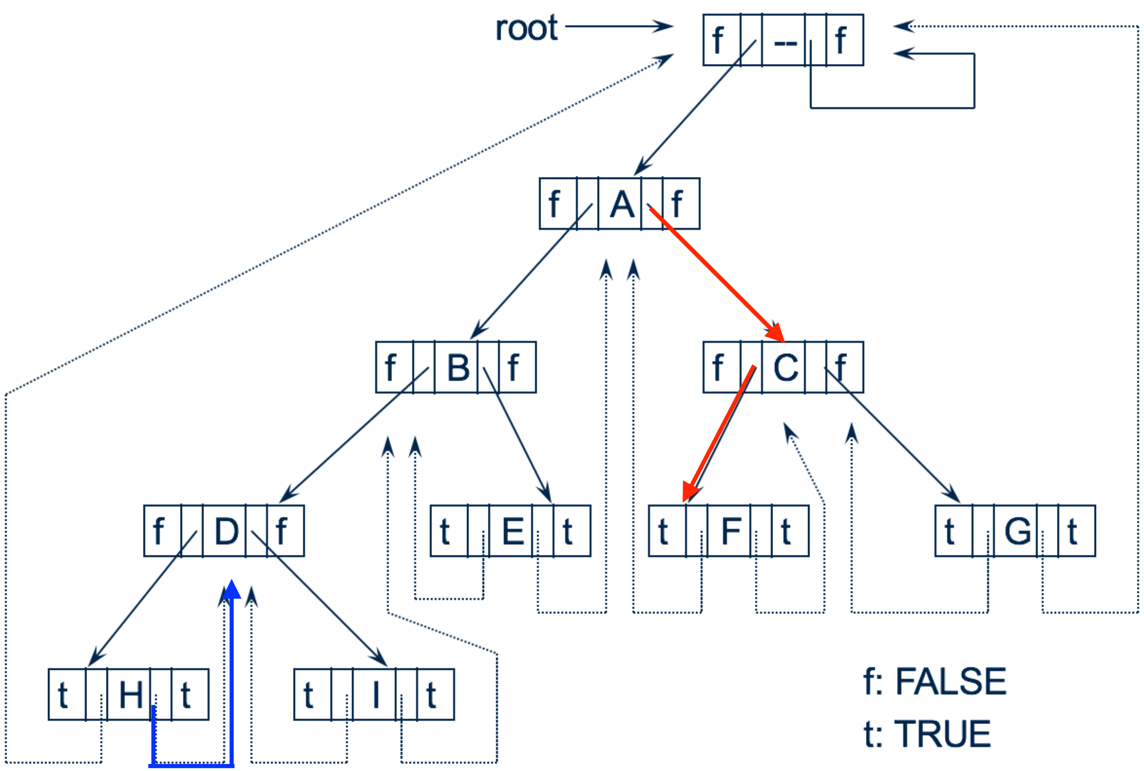


in-order traversal of threaded binary trees

- find the in-order successor of ptr without using stack
  - if ptr -> right_thread = TRUE, ptr -> right_child
  - otherwise follow a path of left_child links from the right_child of ptr until we reach a node with left_thread = TRUE

```
threaded_ptr insucc(threaded_ptr tree) {
    threaded_ptr temp;
    temp = tree->right_child;
    if (!tree->right_thread)
        while (!temp->left_thread)
            temp = temp->left_child;
    return temp;
}
```



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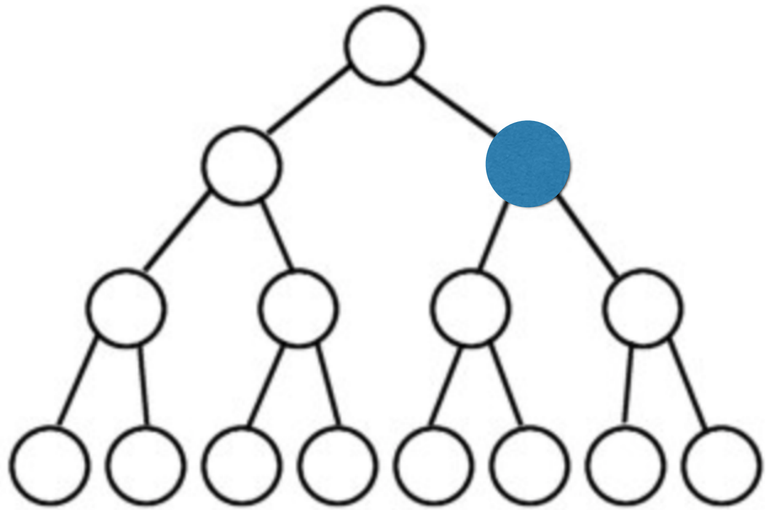

Which node will be returned if blue node is passed into the function insucc?

in-order traversal of threaded binary trees

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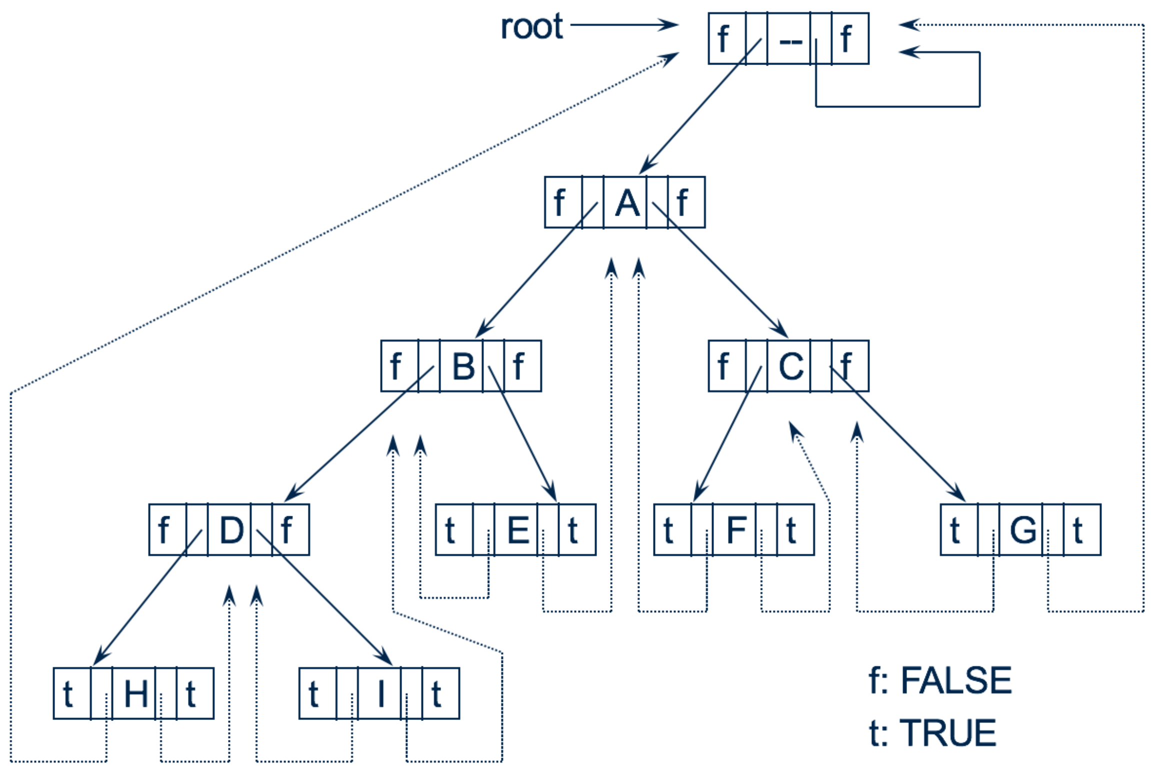
```



Which node will be returned if root node is passed into the function insucc?

in-order traversal of threaded binary trees

```
void tinorder(threaded_ptr tree) {
        threaded_ptr temp = tree;
        for (;;) {
                temp = insucc(temp);
                if (temp == tree) break;
                printf("%3c", temp->data);
        }
}
```